# Systems of Linear Equations

Definition:
A system of equations is a set of two or more equations in two or more variables.

Consider the system below:

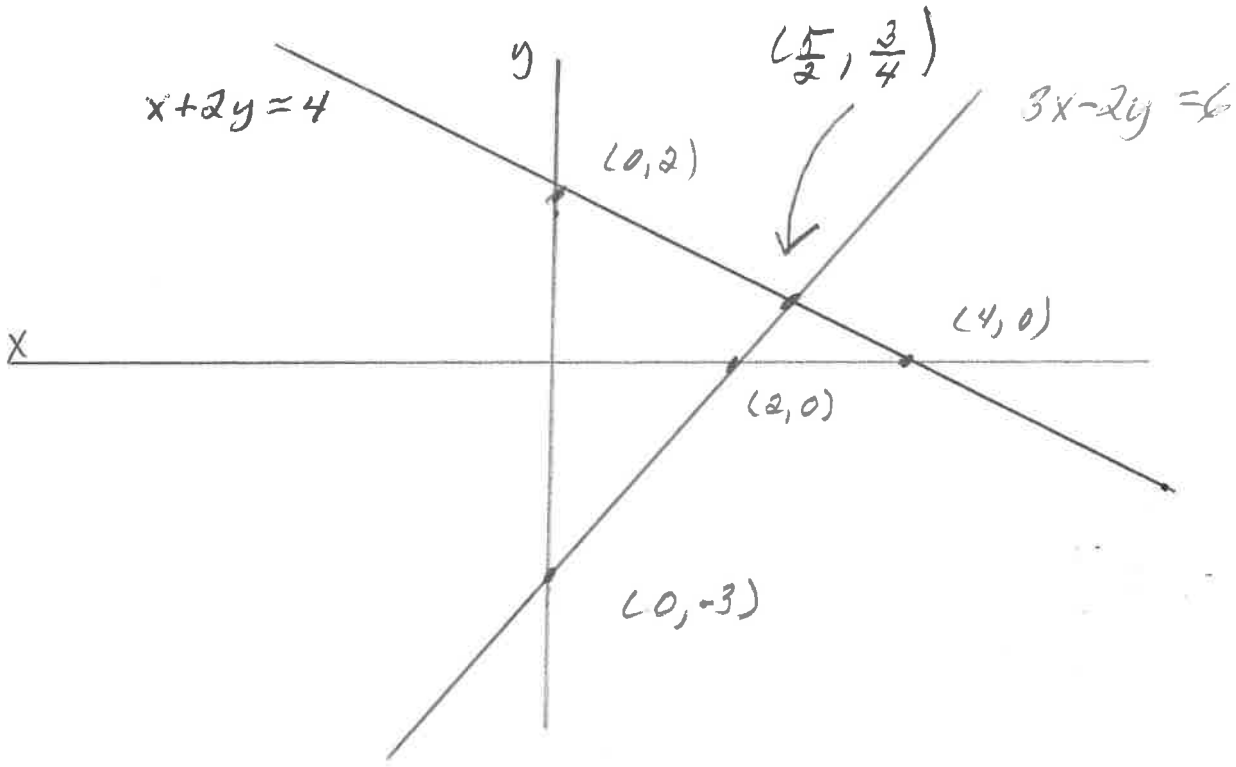
$x + 2y = 4$

$3x - 2y = 6$

What does it mean to be a solution to this system?

The point (a, b) is a solution provided it satisfies both equations. It lies on both lines.

Solve the system above using a graph.



Review:
Solve the system using elimination and substitution:

$4x + 9y = -74$
$8x + 7y = -82$

$-8x - 18y = 148$
$8x + 7y = -82$

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$-11y = 66$

$y = -6$

$4x + 9(-6) = -74$

$4x - 54 = -74$

$4x = -74 + 54$

$4x = -20$

$x = -5$

$(-5, -6)$

Solve the system:

$x - 5y = -5$
$3x + 15y = 17$

$-3x + 15y = 15$
$3x + 15y = 17$

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$30y = 32$, $y = \frac{32}{30}$, $y = \frac{16}{15}$

$x - 5\left(\frac{16}{15}\right) = -5$

$x - \frac{16}{3} = -5$, $x = -5 + \frac{16}{3} = \frac{-15+16}{3}$

$x = \frac{1}{3}$

Solve the system:

$x + 5y = 12$
$-4x - 20y = -48$

$4x + 20y = 48$
$-4x - 20y = -48$

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$0 = 0$

Solution: $\{(x,y) : x+5y = 12\}$

All points on the line $x+5y = 12$

Solve the system:

$$\begin{cases} x + 3y = 2 \\ y - 4z = -13 \\ -3x - 5y + 3z = -19 \end{cases}$$

$x + 3y = 2$

$y - 4z = -13$

$x + 3y = 2$

$-3y + 12z = 39$

$x + 12z = 41$

$y - 4z = -13$

$y = 4z - 13$

$-3x - 5(4z - 13) + 3z = -19$

$-3x - 20z + 65 + 3z = -19$

$-3x - 17z = -84$

$3x + 36z = 123$

$-3x - 17z = -84$

$19z = 39$

$z = \frac{39}{19}$

$y = 4\left(\frac{39}{19}\right) - 13 = -\frac{91}{19}$, $y = -\frac{91}{19}$

$x = 2 - 3y = 2 - 3\left(-\frac{91}{19}\right)$, $x = \frac{311}{19}$

Solve the system:

$$\begin{cases} -3x+3y+z=2 \\ x+6y-4z=-11 \\ 4x+3y-5z=-13 \end{cases}$$

$-3x+3y+z=2$

$3x+18y-12z=-33$

$21y-11z=-31$

$-4x-24y+16z=44$

$4x+3y-5z=-13$

$-21y+11z=31$ or $21y-11z=31$

Let $z=k$, $21y-11k=-31$

$y=\frac{11k-31}{21}$, $y=\frac{11}{21}k-\frac{31}{21}$

$x+6\left(\frac{11k-31}{21}\right)-4k=-11$

$x+\frac{66k}{21}-\frac{186}{21}-4k=-11$

$x=4k-\frac{66}{21}k+\frac{186}{21}-11$

$x=\frac{18}{21}k-\frac{45}{21}$

Solve the system:

$$\begin{pmatrix} 4x - 24y + 8z = 9 \\ 3x + 5y - 5z = 19 \\ x - 6y + 2z = 3 \end{pmatrix}$$

$4x - 24y + 8z = 9$

$-4x + 24y - 8z = -12$

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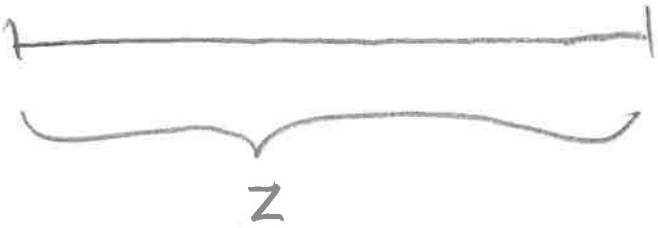
$0 = -3$

No solution!

A private jet flies the same distance in 7 hours that a commercial jet flies in 5 hours. If the speed of the commercial jet was 126 mph less than two times the speed of the private jet, find the speed of each jet.

distance = rate x time




Let $y$ = speed of the private jet

$x$ = speed of the commercial jet.

$$\left.\begin{aligned} x &= 2y - 126 \\ 7y &= z \\ 5x &= z \end{aligned}\right\}$$

$x = 2y - 126$

$7y - 5x = 0$

$7y = 5x$, $y = \frac{5}{7}x$

$x = 2\left(\frac{5}{7}x\right) - 126$, $\frac{10x}{7} - x = 126$

$10x - 7x = 882$, $3x = 882$

$x = 294$ mph, $y = \frac{5}{7}(294)$

$y = 210$ mph

$z = 1470$ miles

How many ounces of a 16% acid solution and a 32% acid solution must be combined to obtain 74 ounces of 24% acid solution?

Let $x$ = amount of 16% acid solution

$y$ = amount of 32% acid solution

$$\begin{cases} x + y = 74 \\ (.16)x + (.32)y = (.24)74 \end{cases}$$

$$\begin{aligned} x + y &= 74 \\ 16x + 32y &= 1776 \\ -16x - 16y &= -1184 \\ \hline 16y &= 592 \end{aligned}$$

$y = 37$ ounces

$x = 74 - y = 74 - 37 = 37$ ounces

Applications:

1. understan
2. choose variable(s)
3. describe application mathematically
4. do the math
5. interpret results